Research reactor simulator

PHYSICS MANUAL
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1. Introduction
The Research Reactor Simulator, developed by Jozef Stefan Institute (IJS) is a real-time simulation code of a Triga-type reactor in real time. The simulator is documented in three manuals:

- **User manual** – Describing the operation of the reactor simulator from the user’s perspective
- **Physics manual** – Describing the physics models implemented in the simulator
- **Developers manual** (not yet released) – Description of the software architecture with instructions for compiling and running the code

This document is the Physics manual for the Research Reactor Simulator.

2. Physics models

2.1. Reactor kinetic

The time behaviour of neutron population in a nuclear reactor can be described by point kinetic equations [1]

\[ \frac{dN(t)}{dt} = \rho(t) - \frac{\beta_{\text{eff}}}{l} N(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \]

\[ \frac{dC_i(t)}{dt} = \frac{\beta_i}{l} N(t) - \lambda_i C_i(t) \quad i = 1, \ldots, 6. \]

\( N(t) \) represents the total, time dependent population of neutrons in the reactor core and \( C_i(t) \) represent neutron populations of individual groups. \( l = 39.5 \times 10^{-5} \text{s} \) and \( \beta_{\text{eff}} \) denote prompt neutron lifetime and effective delayed neutron fraction, respectively. The delayed neutron fractions and the decay constants for individual groups are denoted by \( \beta_i \) and \( \lambda_i \), respectively and available from the source code of the digital reactivity meter [8]. The decay constants \( \lambda_i \) are expressed in sec\(^{-1}\) and connected to the mean time \( \tau_i \) as \( \tau_i = 1/\lambda_i \).

Delayed neutron fractions \( \beta_i \) are directly used as a sum to calculate \( \beta_{\text{eff}} \). Time dependent reactivity \( \rho(t) \) is calculated as

\[ \rho(t) = \rho_0(t) - \alpha(T(t)) T(t) - \rho_{\text{Xe}}(t) , \]

where \( \rho_0 \) is the reactivity of a cold reactor core free of any neutron poison, calculated based on control rod positions, \( \alpha(T) \) is temperature dependent fuel temperature feedback coefficient and \( \rho_{\text{Xe}}(t) \) is reactivity change due to \( ^{135}\text{Xe} \) poisoning.

The delayed neutron populations \( C_i(t) \) are plotted on a separate graph for all six groups. The simulator offers an option to disregard the delayed neutron production by setting \( \beta_i = 0, i = 1, \ldots, 6 \). The time evaluation of the kinetic equations (1) is an iterative process with a fixed step size \( dt \).
Each step consists of the following operations

1. Evaluation of the kinetic equations (1)
2. Evolution of the fuel temperature $T(t)$ and water temperature $T_w(t)$
3. Time propagation of the xenon concentration $Xe(t)$

Point kinetic equations are solved using an explicit, forth order Runge-Kutta solver with a fixed step size [2]. One equation is evaluated to propagate the neutron population $N(t)$ and another six equations to evaluate the neutron concentration for each individual group $\tilde{C_i}$.

### 2.2. Temperature feedback effects

The fuel elements are being heated by fission and heated or cooled by temperature conduction between the fuel and the surrounding water. The cooling water gains heat from the fuel elements and loses it to the surroundings due to active cooling, that can be turned on or off in the simulator. While heat originating from the fuel elements from fission can be calculated from the reactor power, cooling of the fuel elements is a non-trivial problem connected to thermodynamic processes and could not be modelled analytically with satisfactory precision in one dimension. It should be noted that proper description of heat transfer and temperature profiles in the core can be calculated only by using 3D computational fluid dynamics simulations [6], which are beyond the scope of this real-time simulator. Hence approximations had to be used.

Temperature transitions are implemented using an asymptotic formula:

$$T_{fe}^{'} = T_{fe} + dt \frac{P(t)}{C_f} (T_{fe}) - T_{cooling},$$

(3)

$T_{fe}$ and $T_{fe}^{'}$ are fuel element temperatures at previous and next iteration and $\alpha$ is a constant fitted to experimental data in units ofpcm/°C.

The cooling temperature $T_{cooling}$ is modelled using a semi empirical polynomial of the third order which is used by the TRIGLAW code [7] for calculation of temperature of typical TRIGA fuel elements cooled by natural convection

$$T_{fe \ stat} = a_1 P_{el} + a_2 P_{el}^2 + a_3 P_{el}^3 + T_w,$$

(4)

where $T_{fe \ stat}$ is the temperature of the fuel element in the stationary conditions, $P_{el}$ is the power produced at each fuel element, $a_1$, $a_2$ and $a_3$ are constants fitted to experimental data and $T_w$ is the temperature of the cooling water. Assuming that all power released in the fuel leaves the system in stationary conditions, the polynomial (4) is reversed to calculate $P_{el}$, which can be used to calculate:

$$T_{cooling} = \frac{P_{el}}{C_f}. $$

(5)
The fuel heat capacity $C_f$ is calculated using the formula 5.4 from [4]:

$$c_p(t) = 333 \frac{J}{kgK} + 0.678 \frac{J}{kgK^2} (T_{fe}[K] - 273K).$$  \hfill (6)

The cooling water is heated by the fuel elements and cooled by conduction to the surroundings and the active cooling.

$$T_w' = T_w + \frac{P dt}{C_w} - (T_w - T_s) \beta dt - Adt$$  \hfill (7)

The positive term $\frac{P dt}{C_w}$ is calculated under the assumption that all the heat generated by the fuel elements ends up in the cooling water. $dt$ defines the step size and $C_w$ is the heat capacity of the cooling water expressed in the units of J/K and calculated from the water volume $V_w$ as $C_w = V_w \rho_w c_w$. Water volume $V_w$ is approximated from the technical drawings, available on [3], water density $\rho_w = 998.2 \text{ kg/m}^3$ and specific heat capacity $c_w = 4.185 \text{ kJ/(kg K)}$ at 20°C were taken from [4].

The term $(T_w - T_s) \beta dt$ describes the heat that is lost to the environment due to heat conduction. The cooling water surroundings are treated as a reservoir with infinite heat capacity and constant temperature $T_s = 20 \text{ °C}$, based on a temperature measurement inside the reactor hall. The constant $\beta = 0.000013$ is calculated from temperature measurements using linear regression [5]. It is assumed that active cooling removes a constant amount of heat from the system and lowers the temperature for $Adt$ each step until water reaches room temperature.

The effect of fuel temperature on reactivity $\rho_T$ is calculated using a temperature dependent feedback coefficient $\alpha_T(T)$ as

$$\rho_T = \alpha_T(T) T.$$  \hfill (8)

Mimicking the measured temperature dependence measured at the JSI TRIGA, a function consisting of two straight lines with adjustable parameters is used

$$\alpha_T(T) = \begin{cases} aT + b; & aT + b > cT + d \ 
cT + d; & aT + b < cT + d' \end{cases}$$  \hfill (9)

The parameters $a, b, c, d, d'$ are chosen by the simulator user.
2.3. Xenon poisoning

The RR simulator simulates poisoning by $^{135}$Xe with a cross section for neutron absorption of $2.65 \times 10^6$ barn. Simulation of poisoning by the strongest contributor $^{149}$Sm with the cross section lower by two orders of magnitude is not yet implemented. $^{136}$Xe is generated directly from uranium fission and from the decay of $^{135}$I. The iodine is generated from decay of some short lived products. The $^{135}$I production is simulated with a differential equation under the assumption that it is generated directly from uranium [1]:

$$\frac{\partial I}{\partial t} = \gamma_I \Sigma_f \Phi(r, t) - \lambda_I I(r, t),$$

(10)

where $\lambda_I$ is the $\beta$ decay constant for iodine, $\gamma_I$ I the iodine fission yield and $\Sigma_f$ the macroscopic cross section for fission. With the iodine concentration under control, the differential equation for changes in the $^{135}$Xe concentration can be written:

$$\frac{\partial X}{\partial t} = \gamma_X X(r, t) \Phi(r, t) + \lambda_I I(r, t) - \lambda_X X(r, t) - \sigma_{\alpha}^X X(r, t) \Phi(r, t).$$

(11)

$\gamma_X$ and $\lambda_I$ are the xenon generation probability and the xenon decay constant, $\sigma_{\alpha}^X$ is the microscopic cross section for absorption. Equations (10) and (11) are solved each computation step using the a first order solver of differential equations. Since changes of the poison concentrations in the reactor core occur on a much slower time scale than the changes in neutron concentrations, using a fourth order differential equations solver would introduce unnecessary computational overhead with such a small step size. The step size was kept constant to preserve a consistent number of data points for all time dependent physical quantities and avoid the need for interpolation between points in time.

The effect of $^{135}$Xe on reactivity is calculated as by

$$\Delta \rho_{xe} = \frac{\sigma_{\alpha}^X X(t)}{\Sigma_f}. $$

(12)
2.4. Constant power initial conditions

To make the reactor power constant, it is also necessary to set the number of neutrons in all six groups and the total number of neutrons to values that allow stationary conditions. Such values are calculated by setting the derivatives on the left side of the point kinetics equations (1) to zero and solving the equations for $N(t)$ and $C_i$. The resulting equations can be written out as

\[
N(t) = \frac{P(t)c}{\Sigma_{\text{total}} \mu_{\text{avg}}}
\]

\[
C_i = \frac{N(t)\beta_i}{\lambda_i t}.
\]  

(13)

2.5. Operation limits

The operation limits are checked every iteration of the simulation so the emergency shutdown signals can be triggered when necessary. The power scram signal, fuel temperature scram signal and water temperature scram signal shutdown the reactor when the corresponding values surpass limit values that are settable from the user interface. The period scram only activates only, if the calculated reactor period surpasses the limit value for more than a certain amount of time. The reactor period is numerically calculated as an exponentially weighted moving average of $N_T$ points. The moving average is calculated as

\[
T_s = \sum_{i=1}^{N_T} T_k^N_{T_i - i - 2} T_i T_w.
\]

(14)

Where $T_i$ is the reactor period corresponding to one calculation cycle, computed as

\[
T_i = \frac{dt}{\log N_{i+1} - N_i}.
\]

(15)

$T_w$ is the sum of weights, that represent a geometric series, computed as

\[
T_w = \frac{1 - T_k^N_{T - 2}}{T_k}.
\]

(16)

$T_k$ is a weightening factor. Automatic emergency shutdown 6 seconds after the reactor pulse was also implemented to mimic the behaviour of IJS TRIGA.
3. REFERENCES

5. Štancar, Ž. Snoj, L, An improved thermal power calibration method at the TRIGA Mark II